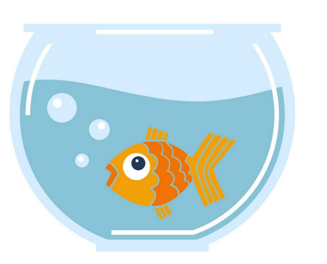
Games, graphs, and machines



September 27, 2024

Distinguishable strings

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Say that L *distinguishes* x and y if there exists a z such that exactly one of xz or yz is in L.

Distinguishable strings

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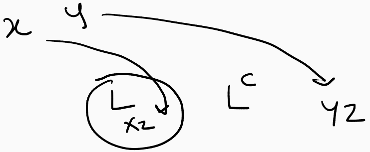
Example

Let $L = 01^*0|10^*1$. $= \; 00, 010, 0110, 01110, \ldots$
$\quad 11, 101, 1001, 10001, \ldots$

1. Does L distinguish 0 and 1? $z = 10$, $xz = 010$ ✓ $yz = 110$ ✗
2. What about 01 and 10? $z = 0$ or 1 or 10
3. $\underbrace{\text{What about 010 and 101?}}_{\text{NO.}}$



Indistinguishable strings

Say that $x \sim_L y$ if L cannot distinguish x and y.

Proposition: $\sim_L$ is an equivalence relation.

$x \sim y \iff$ for all z

xz , yz both in L

both not in L

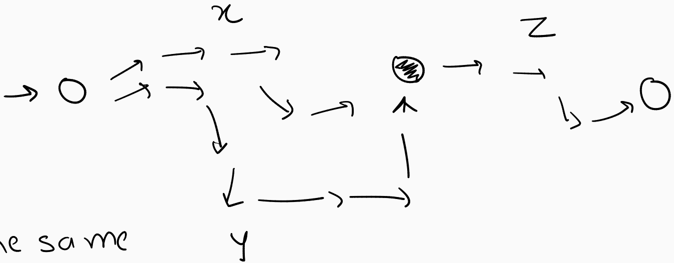
How do we know $x \sim y$?

Suppose L has a DFA M.

Proposition: If x and y end at the same state in M, then $x \sim_L y$.

xz
yz

end up at the same state!



How do we know $x \sim y$?

Suppose L has a DFA M.

Proposition: If x and y end at the same state in M, then $x \sim_L y$.

Proposition: The number of $\sim_L$ equivalence classes is at most the number of states of M.

Example

Let $L = \{\text{Palindromes}\}$.

$w = w^{rev}$

abba

bob

racecar

Proposition: $01, 001, 0001, 00001, \cdots$ are distinguishable.

010

0010

000010000

00000000010000

so inf many eqv. classes.

Example

Let $L = \{\text{Palindromes}\}$.

Proposition: $01, 001, 0001, 00001, \cdots$ are distinguishable.

Consequence: There is no DFA for L.

The Myhill-Nerode Theorem

Theorem: L is regular if and only if $\sim_L$ has finitely many equivalence classes.

finitely many eqv. cl $\Rightarrow$ DFA / regex

$\boxed{\text{inf. many} \Rightarrow \text{NO DFA / regex}}$

$L = \{0^n 1^n \mid n \geq 0\}$

Not regular.

$\Uparrow$

$$\left.\begin{array}{l} 01 \\ 001 \\ 0001 \\ 00001 \\ 000001 \\ \vdots \end{array}\right\}$$

distinguishable